Version 1.0

General Certificate of Education (A-level)
June 2012

Mathematics MFP1

(Specification 6360)

Further Pure 1

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

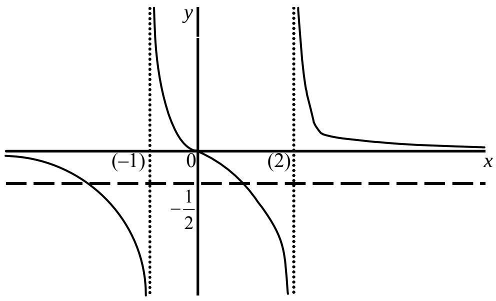
Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
				Accept correct equivalent decimals in place of some/all fractions in the scheme
1(a)	$\alpha+\beta=\frac{7}{5}\ (=1.4)$	B1		
	$\alpha\beta=\frac{1}{5}\ (=0.2)$	B1	2	
(b)	$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$	M1		OE eg $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\frac{1}{5}\left[7(\alpha+\beta)-1-1\right]}{\alpha\beta}$ scores M1 m1
	$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\quad=\frac{\left(\frac{7}{5}\right)^2-2\left(\frac{1}{5}\right)}{\frac{1}{5}}$	m1		Correct expression for $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ in terms of either $(\alpha+\beta)$ and $\alpha\beta$ or with numerical substitution of correct/c's values from (a)
	$=\frac{\frac{49}{25}-2\left(\frac{1}{5}\right)}{\frac{1}{5}}\ =\frac{\frac{49}{25}-\frac{2}{5}}{\frac{1}{5}}=\frac{\frac{39}{25}}{\frac{1}{5}}=\frac{39}{5}$	A1	3	CSO AG must see some intermediate evaluation, must see one of the first three expressions A0 if $\alpha+\beta$ has wrong sign
(c)	(Sum=) $\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}\ =\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$ $\left(=\frac{7}{5}+\frac{\frac{7}{5}}{\frac{1}{5}}\right)$	M1		Writing $\alpha+\frac{1}{\alpha}+\beta+\frac{1}{\beta}$ in a correct suitable form or with numerical values
	(Product =) $\alpha\beta+\frac{\alpha}{\beta}+\frac{\beta}{\alpha}+\frac{1}{\alpha\beta}$ $=\frac{1}{5}+\frac{39}{5}+5$	M1		Correct expression for product into which substitution of numbers attempted for all terms, at least one either correct/correct ft
	Sum = $\frac{42}{5}$, Product = 13	A1		OE <u>Both</u> **SC** If B0 for $\alpha+\beta=-\frac{7}{5}$ in (a), and (c) S= $-\frac{42}{5}$ oe, P = 13 award this A1
	$x^2-Sx+P\ (=0)$	M1		Using correct general form of LHS of equation **with** ft substitution of c's S and P values. PI. M0 if either $S=\alpha+\beta$ or $P=\alpha\beta$ values
	Equation is $5x^2-42x+65=0$	A1	5	CSO Integer coefficients and '= 0' needed. Dependent on B1B1 in (a) and previous 4 marks in (c) scored
	Total		**10**	

Q	Solution	Marks	Total	Comments
2(a)	$y = x^4 + x$			
	$\{y(-2+h) =\}\quad (-2+h)^4 + (-2+h)$	M1		$(-2+h)^4 + (-2+h)$ PI
	$= h^4 - 8h^3 + 24h^2 - 32h + 16 - 2 + h$	B1		Correct expansion of $(-2+h)^4$ as $h^4 - 8h^3 + 24h^2 - 32h + 16$ PI
	$= h^4 - 8h^3 + 24h^2 - 31h + 14$	A1F		Seen separately or as part of the gradient expression. Ft one incorrect term in expansion of $(-2+h)^4$
	Gradient = $\dfrac{y_2 - y_1}{x_2 - x_1}$			
	$= \dfrac{h^4 - 8h^3 + 24h^2 - 31h + 14 - (14)}{-2 + h - (-2)}$	M1		Use of correct formula for gradient PI
	$= \dfrac{h^4 - 8h^3 + 24h^2 - 31h}{h} =$ $h^3 - 8h^2 + 24h - 31$	A1	5	The four correct terms in any order A0 if incorrect (constant/h) term ignored due printed form of answer
(b)	As $h \to 0$, gradient of line in **(a)** $\to$ gradient of curve at point (−2, 14)}	E1		Lim [c's($p+qh+rh^2+h^3$)] OE $h \to 0$ NB '$h=0$' instead of '$h \to 0$' gets E0
	{Gradient of curve at point (−2, 14) is} −31	E1	2	Dependent on previous E1 and printed **form** of answer in (a) obtained convincingly but then ft on c's p value
	Total		**7**	
3(a)	$\mathrm{i}(z+7) + 3(z^* - \mathrm{i}) =$ $\mathrm{i}(x + \mathrm{i}y + 7) + 3(x - \mathrm{i}y - \mathrm{i})$	M1		M1 for use of $z^* = x - \mathrm{i}y$
	$= \mathrm{i}x - y + 7\mathrm{i} + 3x - 3\mathrm{i}y - 3\mathrm{i}$	M1		M1 for $\mathrm{i}^2 y = -y$
	$= 3x - y + \mathrm{i}(x - 3y + 4)$	A1	3	If the five terms correct but not grouped into Real and Imaginary parts, allow A1 retrospectively provided the correct two expressions used in the M1 line in (b)
(b)	$3x - y = 0,\quad x - 3y + 4 = 0$	M1		Attempting to equate all Real parts to zero and all Imaginary parts to zero
	$x - 9x + 4 = 0$ (or eg $y - 9y + 12 = 0$)	A1		A correct equation in either x or y PI by correct final answer
	Solving to give $z = \dfrac{1}{2} + \dfrac{3}{2}\,\mathrm{i}$	A1	3	Allow $x = \dfrac{1}{2},\ y = \dfrac{3}{2}$
	Total		**6**	

Q	Solution	Marks	Total	Comments
				Watch out for the many correct different forms of the general solutions
4	$\sin\left(70^\circ - \frac{2}{3}x\right) = \cos 20^\circ = \sin 70^\circ$	B1		OE cos20 = sin70; or cos20 = sin110 etc PI
	$\sin\left(70^\circ - \frac{2}{3}x\right) = \sin 110^\circ$	B1		OE; Use of a correct angle, in degrees, in other relevant quadrant PI
	$70^\circ - \frac{2}{3}x = 360n^\circ + "70^\circ"$ $70^\circ - \frac{2}{3}x = 360n^\circ + "110^\circ"$	M1		OE; Either one, showing a correct use of 360*n* in forming a general solution. Condone $2n\pi$ in place of 360*n*
	$x = \frac{3}{2}\left(70^\circ - 70^\circ - 360n^\circ\right)$ $x = \frac{3}{2}\left(70^\circ - 110^\circ - 360n^\circ\right)$	m1		Rearrangement of $70 - \frac{2}{3}x = 360n + \alpha$ OE to $x = -\frac{3}{2}\left(\pm 360n + \alpha - 70\right)$ OE, where α is from c's $\sin\alpha = \cos 20$ Condone $2n\pi$ in place of 360*n*
	$x = -540n^\circ$; $x = -540n^\circ - 60^\circ$	A2,1,0	6	OE eg $540n^\circ$, $540n^\circ - 60^\circ$. Condone $0 \pm 540n$ for $\pm 540n$. If not A2, award (i) A1 for either correct unsimplified full general solution or (ii) A1F for correct ft full general solution, ft c's wrong angle(s) after award of B0, may be left in unsimplified form(s) or (iii) A1 for 'correct' simplified full general solution but with radians present A0 for only a partial correct solution
	Total		**6**	

Q	Solution	Marks	Total	Comments
5(a)	Asymptotes			
	$x = -1$	B1		$x = -1$ OE
	$x = 2$	B1		$x = 2$ OE
	$y = 0$	B1	3	$y = 0$
(b)	$-\frac{1}{2} = \frac{x}{x^2 - x - 2} \Rightarrow x^2 - x - 2 = -2x$	M1		Correctly removing brackets and fractions to reach $x^2 - x - 2 = -2x$ OE
	$x^2 + x - 2 = 0 \Rightarrow x = 1,\ x = -2$	A1	2	Correct two values for x-coordinates. NMS 2 or 0 marks
(c)		M1		Three branches shown on sketch of C with either middle branch or outer two branches correct in shape
		A1		All three branches, correct shape and positions and approaching correct asymptotes in a correct manner. If middle branch does **clearly** not go through the origin, then A0
		B1	3	Correct sketch of line (L), $y = -0.5$ identified
(d)	$-2 \le x < -1$	B1		Condone < for ≤ or vice versa
	$1 \le x < 2$	B1		Condone < for ≤ or vice versa
	$-2 \le x < -1,\ 1 \le x < 2$	B1	3	All complete and correct
	Total		**11**	

Q	Solution	Marks	Total	Comments
6(a)	$\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$	M1 A1	2	If A1 not scored, award M1A0 for all correct entries expressed in trig form eg $\begin{bmatrix} \cos 135 & -\sin 135 \\ \sin 135 & \cos 135 \end{bmatrix}$
(b)(i)	$\mathbf{M}=\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} = \sqrt{2}\times\begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$ $= \left(= \begin{bmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{bmatrix} \right)$	M1		Or better PI by cand. having both a correct scale factor of enlargement and a correct corresponding angle of rotation
	Scale factor of enlargement is $\sqrt{2}$	A1		SF = $\sqrt{2}$ OE surd form
	Angle of rotation is 135 (degrees anticlockwise)	A1	3	Angle = 135 OE eg −225 If M0 give B1 for SF=$\sqrt{2}$ OE surd and B1 for angle = 135 OE
(b)(ii)	For $\mathbf{M}^2$, SF of enlargement = 2	B1F		OE If incorrect, ft on [c's SF in (b)(i)]^2
	Angle of rotation is 270 (degrees anticlockwise)	B1F	2	OE, eg − 90(degrees), eg 90 (degrees) clockwise If incorrect, ft on 2×c's angle in (b)(i) (neither B1F B1 nor B1 B1F is possible)
(iii)	$\mathbf{M}^2 = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$ $\mathbf{M}^4 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$	M1		For complete method (matrix calculation or geometrical reasoning) Matrix for $\mathbf{M}^2$ could be seen earlier (M0 if >1 independent error in matrix multiplication) Geometrically SF = 4, rotation angle= 540 OE scores M1 and completion scores A1
	$\mathbf{M}^4 = -4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -4\mathbf{I}$	A1	2	Either of these two forms convincingly shown
(iv)	$\mathbf{M}^{2012} = (\mathbf{M}^4)^{503} = (-4\mathbf{I})^{503} =$ $-(2^2)^{503}\mathbf{I} = -2^{1006}\mathbf{I}$	E1		OE Fully explained, algebraically from $(-4\mathbf{I})^{503}$, or geometrically
	$\mathbf{M}^{2012} = -2^{1006}\mathbf{I}$ (Geometrically: $\mathbf{M}^{2012}$ represents an enlargement with SF 2^{1006} followed by a rotation of angle 2012×135º ie 754.5 revolutions, being equivalent to rotation of 180º ie matrix is $-\mathbf{I}$ so $\mathbf{M}^{2012} = -2^{1006}\mathbf{I}$)	B1	2	$\mathbf{M}^{2012} = -2^{1006}\mathbf{I}$ (n = 1006) (B0 if FIW)
	Total		**11**	

Q	Solution	Marks	Total	Comments
7(a)	Let f(x) = $24x^3 + 36x^2 + 18x - 5$			
	f(0.1) = −2.816, f(0.2) = 0.232	M1		Both attempted and at least one evaluated correctly to at least 1sf rounded or truncated OE fraction
	Change of sign so α lies between 0.1 and 0.2	A1	2	Need both evaluations correct to above degree of accuracy and 'change of sign OE' and relevant reference to 0.1 and 0.2
(b)	f(0.15) = −1.409 (< 0 so root > 0.15)	M1		f(0.15) considered first
	f(0.175) ≈ −0.619 (< 0 so root > 0.175)	A1		f(0.15) then f(0.175) both evaluated correctly to at least 1sf OE fractions
	α lies between 0.175 and 0.2	A1	3	Dependent on both previous marks gained and no other additional evaluations other than at 0.15 and 0.175
(c)	f′ (x) = $72x^2 + 72x + 18$	B1		PI
	(x_2 =) $0.2 - \dfrac{24(0.2)^3 + 36(0.2)^2 + 18(0.2) - 5}{72(0.2)^2 + 72(0.2) + 18}$	B1 B1		B1 for numerator in correct formula B1 for denominator in correct formula
	= 0.1934 (to 4dp)	B1	4	CAO Must be 0.1934 Do not apply ISW NMS scores 0/4
	Total		**9**	

Q	Solution	Marks	Total	Comments
8(a)	$\left(\pm\sqrt{5}\ ,\ 0\right),\quad \left(0\ ,\ \pm 2\right)$	B2,1	2	If not B2, award B1 if either at least two of these 4 correct pts or if '$x = \pm\sqrt{5}$ and $y = \pm 2$'
8(b)	$\dfrac{(x-p)^2}{5} + \dfrac{y^2}{4} = 1$	M1 A1	2	Replacing x by either $x + p$ or $x - p$ and keeping y unchanged or as $y \pm 0$ ACF
8(c)	$\dfrac{(x-p)^2}{5} + \dfrac{(x+4)^2}{4} = 1$	M1		Substitution into c's (**b**) eqn of $y = x+4$ to eliminate y
	$4(x-p)^2 + 5(x+4)^2 = 4 \times 5$ $4(x^2 - 2px + p^2) + 5(x^2 + 8x + 16) = 20$	m1		Denominators 5 and 4 cleared in a correct manner and at least either a correct expansion of $(x \pm p)^2$ or a correct expansion of $(x+4)^2$
	$4x^2 - 8px + 4p^2 + 5x^2 + 40x + 80 = 20$			
	$9x^2 - (8p - 40)x + 4p^2 + 60 = 0$	A1	3	CSO No errors in any line of working. AG. Must see brackets correctly removed and all terms involving x, p correctly rearranged to same side before the printed answer is stated. Must have '= 0' although brackets around $4p^2 + 60$ may be omitted
(d)	Discriminant is $(8p - 40)^2 - 4\ (9)\ (4p^2 + 60)$	B1		OE Must be isolated, not just within the quadratic formula
	For tangency $(8p - 40)^2 - 4\ (9)\ (4p^2 + 60) = 0$	M1		OE Equating c's discriminant to zero before obtaining any values for p
	$p^2 + 8p + 7 = 0$	A1		ACF with like terms collected
	$\{(p+1)\ (p+7) = 0 \Rightarrow\}\ p = -1, p = -7$ (*)	B1		Correct values $-1, -7$ for p
	$\underline{p = -1}$: $9x^2 + 48x + 64\ (= 0)$ $\underline{p = -7}$: $9x^2 + 96x + 256\ (= 0)$	M1		Substitutes at least one of c's **two** values for p either into the given quadratic in (c) OE or into $\dfrac{8p - 40}{18}$
	$\underline{p = -1}$: $9x^2 + 48x + 64\ (= 0) \Rightarrow x = -\dfrac{8}{3}$	A1		$x = -\dfrac{8}{3}$ OE as only root from the quadratic or from $\dfrac{8p - 40}{18}$. Apply FIW if (*) is B0
	$\underline{p = -7}$: $9x^2 + 96x + 256\ (=0) \Rightarrow x = -\dfrac{16}{3}$	A1		$x = -\dfrac{16}{3}$ OE as only root from the quadratic or from $\dfrac{8p - 40}{18}$. Apply FIW if (*) is B0
	$x = -\dfrac{8}{3},\ y = \dfrac{4}{3};\quad x = -\dfrac{16}{3},\ y = -\dfrac{4}{3}$ $\left(-\dfrac{8}{3}, \dfrac{4}{3}\right)\ \left(-\dfrac{16}{3}, -\dfrac{4}{3}\right)$	A1	8	CSO Previous 7 marks must have been awarded and coordinates of both points need to be correct and exact but accept in either format
	Total		**15**	
	TOTAL		**75**	